**Math 10A**
**Worksheet, Discussion #12; Tuesday, 7/3/2018**
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# 1 Newton's Method

## 1.1 Concepts

1. Newton's method helps us approximate the zeros of a function $f(x)$. It is a recursive process in that we start with some guess $x = x_0$, then use Newton's method to give us a better guess $x_1$, and we can do this over and over again to get better and better guesses. The equation is
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

## 1.2 Problems

2. Use Newton's method with two steps to estimate $\sqrt{5}$.

**Solution:** We want to find the root of $x^2 - 5 = 0$. The first guess is $x = 2$ and the next point is
$$x' = 2 - \frac{-1}{4} = \frac{9}{4}.$$
Doing that again, we get that the next point is
$$x' = \frac{9}{4} - \frac{81/16 - 5}{9/2} \approx 2.2361.$$
The real answer is approximately 2.23607.

3. Use Newton's method to estimate $\sqrt[4]{16.32}$.

**Solution:** This value is a root of $x^4 - 16.32 = 0$. We can start at $x = 2$ and using Newton's method gives us
$$x' = x - \frac{f(x)}{f'(x)} = 2 - \frac{-.32}{4 \cdot 2^3} = 2 + 0.01 = 2.01.$$
The real answer is about 2.0099.

4. Find the critical points of $g(x) = \sin(x) - x^2$

**Solution:** We want to find when the derivative is 0 or when $f(x) = \cos(x) - 2x = 0$. Taking the derivative again, we find that it is $-\sin(x) - 2 < 0$ for all $x$. So this function is always decreasing and has a unique root. We plug in $x = 0$ to start, then calculate

$$x' = x - \frac{f(x)}{f'(x)} = -\frac{1}{-2} = \frac{1}{2}.$$

The real solution is $\approx 0.45$.

5. Use Newton's method to estimate $\sqrt[3]{28}$.

**Solution:** We want to find the root of $x^3 - 28$. We guess $x = 3$ and get that the next point is

$$x' = x - \frac{f(x)}{f'(x)} = 3 - \frac{-1}{27} = \frac{82}{27} \approx 3.037.$$

The real solution is $\approx 3.0366$.

6. Find the critical points of $e^x + x^2$.

**Solution:** The critical points are when the derivative is 0 so $f(x) = e^x + 2x = 0$. Taking the second derivative, we have that $f'(x) = e^x + 2 > 0$ so this is an always increasing function. Therefore, it will only have 1 zero. We plug in the only value that we know of $x = 0$ and get

$$x' = x - \frac{f(x)}{f'(x)} = 0 - \frac{1}{3} = -\frac{1}{3}.$$

The real solution is $\approx -0.35$.

7. Find when $\cos x = x$.

**Solution:** Notice that when taking the derivative of $\cos x - x$, we get $\sin x - 1 \leq 0$ so this is a decreasing function which has at most on zero. We start at $x = 0$ to get the next point

$$x' = x - \frac{f(x)}{f'(x)} = 0 - \frac{1}{-1} = 1.$$

The real solution is $\approx 0.739$.

8. Find the roots of $f(x) = x^3 - x + 1$.

**Solution:** Taking the derivative, we get that the derivative is $3x^2 - 1$. This has roots at $\pm 1/\sqrt{3}$. When we plug in $1/\sqrt{3}$, we get that $f(1/\sqrt{3}) = 1 - 2/3\sqrt{3} > 0$. Thus, this function only has one zero because the local minimum at $x = 1/\sqrt{3}$ is positive. Since $x = -1/\sqrt{3}$ is a local maximum, we know that the zero must be $< -1/\sqrt{3}$. We can start by guessing $x = -2$. The formula for Newton's method gives us

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} = x - \frac{3x^2 - 1}{x^3 - x + 1}.$$

Plugging in $x = -2$ gives us $\frac{-17}{11}$. So the root is approximately $-1.5454$, the real root is $-1.32$.

9. Use Newton's method to estimate $2^{0.1}$.

**Solution:** We can rewrite this as $2^{1/10}$ so we want to find a root of $x^10 - 2 = 0$. Using Newton's method with a guess of 1 gives us

$$x' = 1 - \frac{-1}{10} = 1.1.$$

The real answer is $\approx 1.0718$.